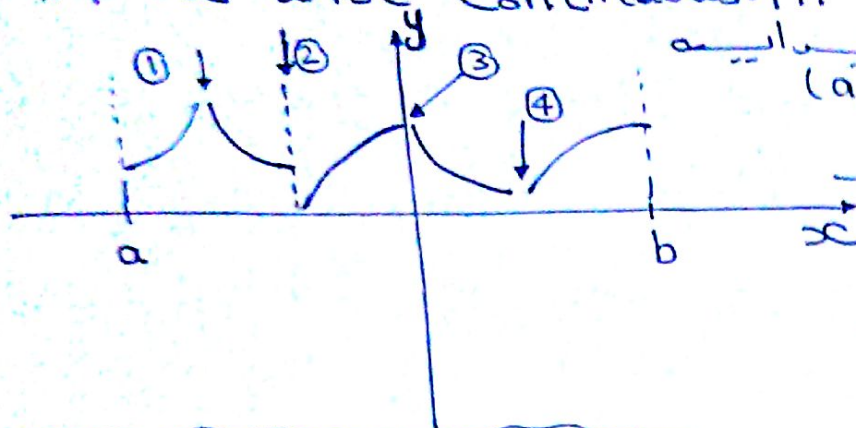


Fowler Series



* Piece wise Continuous. fn



دالة معرفة بالبراييه
والنهاية (a, b)
ويمكن دقة
عند الانتقال

#

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

when Period $P = 2\pi$
where $a_0, a_n, b_n \rightarrow \text{Const}$

$$\int_{-\pi}^{\pi} f(x) dx = \frac{a_0}{2} \int_{-\pi}^{\pi} dx + \sum_{n=1}^{\infty} \left[\int_{-\pi}^{\pi} a_n \cos nx dx + \int_{-\pi}^{\pi} b_n \sin nx dx \right]$$

$$= \frac{a_0}{2} x \Big|_{-\pi}^{\pi} = \frac{a_0}{2} * 2\pi$$

$$= \pi a_0$$

$$\therefore a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx$$

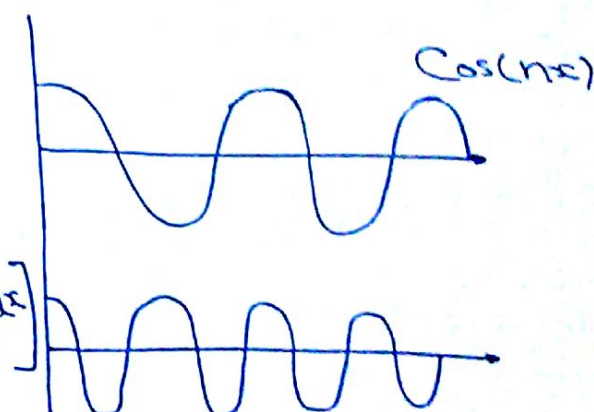
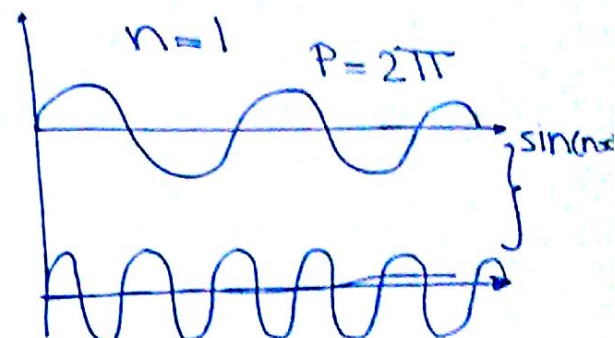
$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx$$

Ref Integration
علاقات مشابهة

#

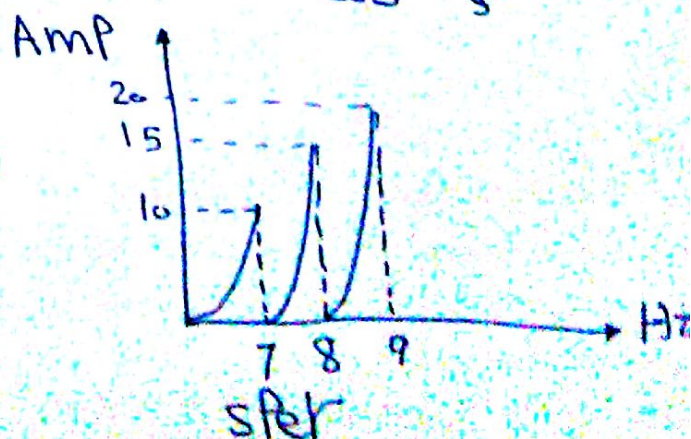
①



by make
Super Position



ينتج من تراكب دالة
sin و cos

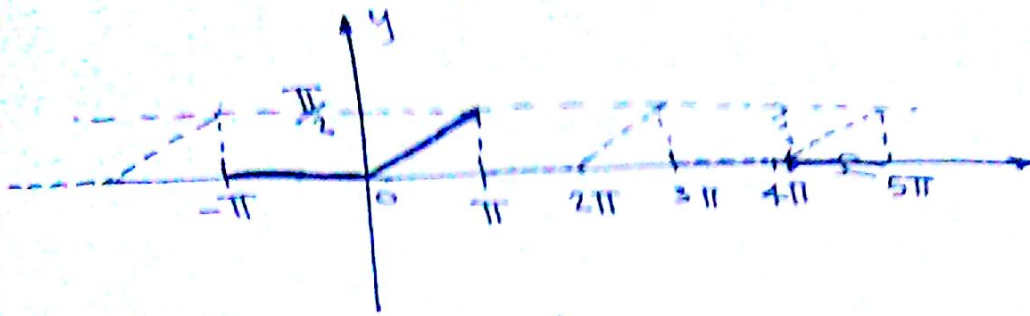


ster

$$f(x) = \begin{cases} 0 & -\pi \leq x < 0 \\ x/2 & 0 \leq x < \pi \end{cases}, f(x+2\pi) = f(x)$$

f_n is periodic

* Plot The function



* by using fourier series

$$\begin{aligned} a_0 &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx = \frac{1}{\pi} \left[\int_{-\pi}^0 0 dx + \int_0^{\pi} \frac{x}{2} dx \right] \\ &= \frac{1}{2\pi} \frac{x^2}{2} = \frac{x^2}{4\pi} \Big|_0^{\pi} \\ &= \frac{\pi^2}{4\pi} = \frac{\pi}{4} \end{aligned}$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$$

$$= \frac{1}{\pi} \left[\int_{-\pi}^0 0 \cos nx dx + \int_0^{\pi} \frac{x}{2} \cos nx dx \right]$$

$$= \frac{1}{\pi} \int_0^{\pi} \frac{x}{2} \cos nx dx = \frac{1}{2\pi} \int_0^{\pi} x \cos nx dx$$

$$\text{Let } u = x \rightarrow du = dx$$

$$dv = \cos nx dx \rightarrow v = \frac{1}{n} \sin nx$$

$$a_n = \frac{1}{2\pi} \left[\frac{x \sin nx}{n} - \int \frac{\sin nx}{n} dx \right]_0^{\pi}$$

$$= \frac{1}{2\pi n} \left[x \sin nx - \int \sin nx dx \right]_0^{\pi}$$

$$= \frac{1}{2n\pi} \left[x \sin nx + \frac{\cos nx}{n} \right]_0^{\pi}$$

$$= \frac{1}{2n^2\pi} (\cos n\pi - \cos 0) = \frac{1}{2n^2\pi} [(-1)^n - 1]$$